The Orbital Motion of The Martian Satellites:

An Application of Artificial Satellite Theory

by

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September 20, 1995

submitted to

6th AAS/AIAA Space Flight Mechanics Meeting
Austin, Texas
February 11-15, 1996

The Martian satellites, Phobos and Deimos, were discovered in 1877 by Asaph Hall. They move in close, nearly circular orbits slightly inclined to the Martian equator. Their principal orbital perturbations are due to the non-spherical nature of the Martian gravity field, but they are also significantly affected by solar perturbations. Planetary perturbations, however, are negligible, and because of their small masses, their mutual perturbations are negligible as well. The studies of their motions from observations have found evidence of a secular acceleration in Phobos' orbit. A similar acceleration in Deimos' orbit is suggested but much less certain. Sharpless (1945) determined such a large value for Phobos' acceleration that it implied an impact with Mars within a few tens of millions of years. One explanation for the source of that acceleration (atmospheric drag on hollow bodies) led to the speculation that the Martian satellites were in fact artificial bodies (Shklovskii and Sagan, 1966). Perhaps it is therefore appropriate to consider employing artificial satellite theory to describe their orbits.

Since their discovery a number of theories have been developed to represent the satellites' orbital motions. These early theories were primarily kinematical in nature, however, Sinclair (1972) created a dynamical theory. Sinclair's work was inspired by the artificial satellite theory of Brouwer (1959) although his variation of parameters approach more closely matched Kozai's (1959, 1962) approach rather than that of Brouwer who favored the von Zeipel technique. Artificial satellite theory has the Earth's (planet's) equator as its reference plane. However, when the Martian equator is used with the Martian satellites, large periodic perturbations due to the Sun appear in the inclinations and nodes, especially for Deimos. Sinclair found that by modifying the theory and referring the elements to the Laplacian planes (Laplace, 1805; Dobrovolskis, 1993), those perturbations could be avoided. The satellite orbits precess almost uniformly on the Laplacian planes because the latter are defined such that the periodic perturbations due to the Martian oblateness and Sun just cancel each other, Included in the theory is a secular acceleration term in the satellites' longitudes to account for any change in the satellites' mean motions. The small observed acceleration for Phobos is currently attributed to the gravitational attraction of the tidal bulge raised by it, 011 Mars. No statistically significant acceleration has yet been observed for Deimos. Sinclair later expanded his theory (Sinclair, 1989) to include a few more terms, especially a previously overlooked long period longitude term pointed out by Born and Duxbury (1975). This term has a 115 km amplitude for Deimos and is an order of magnitude greater than the next largest periodic perturbation. The theory at this point was estimated to have an accuracy Letter than 500 m for Phobos and 1 km for Deimos based on a comparison with numerical integration over 4000" days.

Morley (1990) extended Sinclair's work by increasing the accuracy of the terms describing the secular and periodic perturbations and by adding perturbations due to the tesseral harmonics of the Mars gravity field, short period solar perturbations, and a number of smaller terms. He also extended the theory to second order in the longitude. From a comparison with a 60 day numerical integration, he estimated the theory to be accurate to Letter than 100 m for both satellites.

We have adopted the Sinclair-Morley theory as the basis for Martian satellite ephemerides to be developed in support of J 1'1,'s Mars Exploration Program. The theory 's accuracy is more than sufficient for currently projected spacecraft opera - tions needs. Moreover, its fully analytical nature permits modifications and updates in light of future observations and improvements in the Martian gravity field para m-eters. As part of the initial verification of theory, we compared our implementation to a 60 day numerical integration and found, as did Morley, an accuracy better than 100 meters. We then extended the comparison to 20 years (7300 clays) for Phobos and 80 years (29,220 days) for Deimos. The former comparison covered a time span about 1.8 times the longest period in the Phobos theory, and the latter covered about 1.5 times the longest period in Deimos theory. The long period com-parison suggested that the theories were good to only 300" meters for Phobos and 900 meters for Deimos. Alt hough these accuracies are still adequate for our current purposes, we made an attempt to recover the 100 meter accuracy by modifying the theory. The changes included: (1) basing the Laplacian planes in on mean distance from Mars rather than 011 the semi-~major axis; (2) modifying the computation of the secular perturbations to use the semilatus re ctum instead of semi-major axis and eccentricity; (3) replacing the J_2^2 secular perturbation with that of Kinoshita (1977); (4) adding the J_3^2/J_2, J_2^3, and J_2J_4 secular perturbations from Kinoshita (1977); (5) adding the J_5 and J_7 periodic perturbations from Brouwer (1 959); (6) incorporating additional terms in the J_{22} periodic perturbations (Guinn, 1991); (7) replacing the original long period mean longitude perturbation due to J_2 and the Sun with a more precise expression; (8) adding additional long period solar per-turbation terms. After the changes the Phobos accuracy improved to about 120 meters, Close to our goal. However, the 1)eimos accuracy improved to only about 500 meters. It is limited by a long period mean longitude effect, the source of which is currently unknown, and by insufficient accuracy in the determination of the Laplacian plane inclination.

This paper describes our extended Sinclai r-Morley theory for the Martian satel-lites. It outlines the original theory, presents the modificat ions and additional terms, and compares the theory with numerical integration. The latter comparisons in-clude an examination of the effects of neglected terms in the theory. We also provide a comparison with one of the other competing dynamical Martian satellite theories, namely the E :SAPHO/ESADE theory (Chapornt- Touzé, 1988, 1990a, 1990b). Fi-nally we discuss the limitations of the theory in the context of its use 10 support JPL's Mars Exploration Program.

Acknowledgement

The research described in this paper was performed at the Jet Propulsion Labora-tory, (California Institute of Technology, under contract with the National Aeronau - tics and Space Administration.

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